## Brevia

## SHORT NOTES

# Strain from two angles of shear 

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Abstract-By converting two angles of shear to three relative stretches the shape and orientation of the strain ellipse may be easily calculated.

The shape and orientation of the strain ellipse can be obtained easily from two angles of shear in a plane with a Mohr circle, especially if the pole construction of Lisle (1991, p. 122) is used. Still, an analytical solution would be useful. A direct approach is not possible because the finite shear strain parameter $\gamma^{\prime}$ is defined in terms of both $\gamma$ and $\lambda^{\prime}$. Several previous attempts have been made to find an analytical solution. The approach based on the Mohr circle diagram by Ragan (1980) is clumsy, and the solution of the pair of equations suggested by Ramsay \& Huber (1983, p. 132) is not easy.

If two initially perpendicular lines are deformed they then represent an angle of shear. Following Groshong (1988) these lines can be converted to a relative stretch by a simple construction.

A reference line not parallel to either of the deformed lines is arbitrarily chosen as the $x^{\prime}$ axis and with it an
origin and $y^{\prime}$ axis (Fig. 1a). Two reference points $A^{\prime}(-1,0)$ and $B^{\prime}(+1,0)$ are located on the $x^{\prime}$ axis. A line parallel to one of the deformed lines through $A^{\prime}$ and a line parallel to the other deformed line through $B^{\prime}$ intersect at $P^{\prime}$. As a check, the apex angle $P^{\prime}$ is $90-\psi$. Wellman's (1962) construction shows this point lies on an ellipse with the same shape and orientation as the strain ellipse, and therefore the radius $O P^{\prime}$ is then a relative stretch corresponding to this pair of sheared lines. The co-ordinates of $P^{\prime}$ are obtained from the angle $\alpha^{\prime}$ line $A^{\prime} P^{\prime}$ and the angle $\beta^{\prime}$ line $B^{\prime} P^{\prime}$ makes with the $+x^{\prime}$ axis. Then

$$
\begin{equation*}
\tan \alpha^{\prime}=\frac{y^{\prime}}{x^{\prime}+1} \quad \text { and } \quad \tan \beta^{\prime}=\frac{y^{\prime}}{x^{\prime}-1} . \tag{1}
\end{equation*}
$$

Solving both for $y^{\prime}$ gives

$$
\begin{equation*}
y^{\prime}=\left(x^{\prime}+1\right) \tan \alpha^{\prime} \text { and } y^{\prime}=\left(x^{\prime}-1\right) \tan \beta^{\prime} \tag{2}
\end{equation*}
$$


(a)

(b)

Fig. 1. (a) Construction of a relative stretch (after Groshong 1988). (b) Two deformed brachiopods (after Ragan 1985,

Combining these to eliminate $y^{\prime}$ and rearranging then yields

$$
\begin{equation*}
x^{\prime}=-\frac{\tan \alpha^{\prime}+\tan \beta^{\prime}}{\tan \alpha^{\prime}-\tan \beta^{\prime}} . \tag{3}
\end{equation*}
$$

With $x^{\prime}$ either part of equation (2) can be used to obtain $y^{\prime}$. Then the relative stretch $S$ and its orientational angle $\theta^{\prime}$ are obtained from

$$
\begin{equation*}
S=\sqrt{x^{\prime 2}+y^{\prime 2}} \quad \text { and } \quad \theta^{\prime}=\arctan \left(y^{\prime} / x^{\prime}\right) \tag{4}
\end{equation*}
$$

An example will illustrate the procedure. From the two deformed brachiopods (Fig. 1b), we wish to determine the shape and orientation of the strain ellipse. Relative to the arbitrary co-ordinates axes measurements and calculations give the following results.

|  | $a^{\prime}$ | $\beta^{\prime}$ | $x^{\prime}$ | $y^{\prime}$ | $S$ | $\theta^{\prime}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| a | $70^{\circ}$ | $125^{\circ}$ | -0.31596 | 1.87939 | 1.90576 | $-80.46^{\circ}$ |
| b | $34^{\circ}$ | $99^{\circ}$ | 0.80696 | 1.21881 | 1.46174 | $56.49^{\circ}$ |

These two relative stretches and their corresponding orientation angles together with a third reference stretch $S=1.0$ and angle $\theta^{\prime}=0^{\circ}$ are sufficient for a solution.

Using the matrix method (Ragan 1987), the eigen-
values of the relative finite strain tensor are $\lambda_{1}^{\prime}=0.25060$ and $\lambda_{2}^{\prime}=1.00019$. The strain ratio is then $R_{\mathrm{s}}=\sqrt{\lambda_{2}^{\prime} / \lambda_{1}^{\prime}}=2.0$. The eigenvectors give the orientations of principal relative stretches and they are $\theta_{1}^{\prime}=89.1^{\circ}$ and $\theta_{2}^{\prime}=-0.9^{\circ}$. These results are essentially the same as found graphically (Ragan 1985, p. 197).

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